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THE MEANING OF ROTATION IN THE SPECIAL THEORY OF RELATIVITY

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The novelty of the special theory of relativity consists in its replacing the equations for translation given by the Newtonian theory by the Lorentz transformations. It is natural to ask what new equations should be used to express a rotation in this theory. Most of the writers on the subject (*e. g.*, A. Einstein, *Leipzig, Ann. Physik*, **49**, 1916, §3; M. V. Laue, *Die Relativitätstheorie*, Vol. 2, 1921, p. 162) have assumed that the Newtonian equations for rotation may be used without change in the new theory. While this is permissible as a "first approximation" for points near the axis of rotation, it is not a satisfactory transformation for the whole of space, since it involves velocities greater than that of light for points sufficiently far from the axis. The object of this note is to present a definition of rotation which is free from this objection, and to deduce some of its properties.

In the Newtonian theory the equations of transformation connecting the coördinates in a system K with those in a system K' moving uniformly with respect to K along the x-axis are:

$$x' = x - vt, y' = y, z' = z, t' = t. \tag{1}$$

For a rotation about the z-axis, we have:

$$x' = x\cos\omega t + y\sin\omega t, y' = -x\sin\omega t + y\cos\omega t, z' = z, t' = t, \tag{2}$$

or if polar coördinates are introduced in the xy plane:

$$\theta' = \theta - \omega t, r' = r, z' = z, t' = t. \tag{2a}$$

In the special theory of relativity, in place of (1) we have the Lorentz transformations:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \; y' = y, \; z' = z, \; t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}. \qquad (3)$$

To obtain the corresponding generalization of (2*a*) we shall assume the following properties, which may be used to derive (2*a*) from (1), as characteristic of uniform rotation.

1. The velocity of a point in K' with respect to the point in K with which it momentarily coincides is independent of the time, and is the same for all points at a given distance from the axis of rotation.

2. For the two concentric circles $r' = r = \text{const.}$, the equations of transformation are similar to those for a uniform translation with $r\Theta$ the length of the arc along the circle in place of the linear distance x.

3. The velocity of a point at the distance $r' + \Delta r'$ from the axis with respect to a point at the distance r' from the axis (both in the system K') is a constant (ω) times $\Delta r'$ where $\Delta r'$ is a differential.

From 1 and 2 we find:

$$r'\Theta' = \frac{r\Theta - v(r).t}{\sqrt{1 - v(r)^2/c^2}}, \; r' = r, \; z' = z, \; t' = \frac{t - v(r).r.\Theta/c^2}{\sqrt{1 - v(r)^2/c^2}}. \qquad (4)$$

To utilize 3, we recall the equation for compounding velocities, obtained by differentiating (3): ($dx'/dt' = v_{12}$; $dx/dt = v_2$; $v = v_1$)

$$v_{12} = \frac{v_2 - v_1}{1 - v_1 v_2/c^2}. \qquad (5)$$

For our case we have $v_2 = v(r + \Delta r)$; $v_1 = v(r)$; $v_{12} = \omega\Delta r$, giving:

$$\omega\Delta r = \frac{v(r + \Delta r) - v(r)}{1 - v(r + \Delta r)\, v(r)/c^2}, \qquad (6)$$

or, in the limit,

$$\omega \frac{dr}{dv} = \frac{1}{1 - v^2/c^2}, \qquad (7)$$

and on integration, taking the constant to make $v(0) = 0$:

$$v = c \tanh \omega r/c \qquad (8)$$

By inserting this value in (4), we obtain as our final equations:

$$\Theta' = \Theta \cosh \omega r/c - ct \frac{\sinh \omega r/c}{r}, \; r' = r,$$
$$z' = z, \; t' = t \cosh \omega r/c - r\Theta \frac{\sinh \omega r/c}{c}, \qquad (9)$$

From the form of these equations we see that the totality of transformations corresponding to them forms a group; since the identity is given by setting $\omega = 0$, the inverse of the transformation $\omega = \omega'$ by setting $\omega = -\omega_1$, and the resulting of the two transformations $\omega = \omega_1$ and $\omega = \omega_2$ by setting $\omega = \omega_1 + \omega_2$. We further note that r is restricted to quantities small compared with c, we may replace $\cosh \omega r/c$ by unity and $\sinh \omega r/c$ by $\omega r/c$, and on neglecting the term in r^2/c^2 we obtain (2a). This justifies the use of (2a) as a first approximation to obtain results for actual rotating systems, for which of course r/c is negligibly small. (Cf. the discussion of Sagnac's experiment given by M. V. Laue, *Relativitätstheorie*, Vol. 1, 1919, p. 125 and by L. Silberstein in *J. Optical Soc. Amer.*, **5**, 1921, p. 291 f.)

While the relation between the primed and unprimed coördinates, given by (9) is formally symmetrical, the corresponding spaces are not interchangeable. In fact, if the unprimed space is a "stationary" system the square of the arc element of the space-time continuum is given by:

$$d\sigma^2 - c^2\,dt^2 \tag{10}$$

where $d\sigma$ is the arc element for three-dimensional Euclidean space, and consequently in our coördinates by

$$dz^2 + dr^2 + r^2\,d\Theta^2 - c^2\,dt^2. \tag{11}$$

The corresponding element in the rotating, primed system is obtained by solving (9) and substituting the values in (11). This, of course, leaves the four-dimensional space unchanged; but the three dimensional section obtained by putting t' equal to a constant is no longer the arc element of Euclidean space, but is given by the expression (in which primes are dropped):

$$\begin{aligned} dz^2 + r^2 d\Theta^2 + 2r\,dr\,d\Theta\Big(\omega t - \frac{ct}{r}\sinh\frac{\omega r}{c}\cosh\frac{\omega r}{c} - \Theta\sinh^2\frac{\omega r}{c}\Big) \\ + dr^2\Big(1 - \frac{r^2\omega^2\Theta^2}{c^2} + \omega^2 t^2 + \frac{c^2t^2}{r^2}\sinh^2\frac{\omega r}{c} - \Theta^2\sinh^2\frac{\omega r}{c} \\ - 4\,\omega\Theta t\sinh^2\frac{\omega r}{c} - 2\,\frac{ct^2\omega}{r}\sinh\frac{\omega r}{c}\cosh\frac{\omega r}{c} - 2\frac{r\Theta^2\omega}{c}\sinh\frac{\omega r}{c}\cosh\frac{\omega r}{c}\Big). \end{aligned} \tag{12}$$

To prove that the space with the above element is non-Euclidean, we have merely to show that its Gaussian curvature is not zero. We only cal-

culate it for the case $t' = 0$. For this special case of (12) the curvature is:

$$R = \frac{\frac{\omega^2}{c^2}\left(1 + 2\Theta^2\left(\frac{\omega r}{c} + \sinh\frac{\omega r}{c}\cosh\frac{\omega r}{c}\right)\left(\frac{\omega r}{c}\sinh\frac{\omega r}{c} - \cosh\frac{\omega r}{c}\right)\left(\frac{\cosh\frac{\omega r}{c}}{\frac{\omega r}{c}}\right)\right)}{\left[1 - \Theta^2\left(\frac{\omega r}{c} + \sinh\frac{\omega r}{c}\cosh\frac{\omega r}{c}\right)^2\right]^2} \quad (13)$$

It appears from the calculations just made that the spacial geometry for the rotating system depends on the time and space coördinates of the point considered, and this is at first sight in contradiction with our ordinary ideas of rotating systems. The explanation of this fact is that for a system rotating with respect to a "stationary" system there is no separation into space and time which stands out as the natural one for the entire rotating system, and consequently for this system spacial geometry is only defined when our coördinates have been selected. Since the equations (9) depend on coördinates so chosen that when $t' = 0$, the points on the radius vector $\Theta^1 = 0$ coincide with those on the radius vector $\Theta = 0$, at $t = 0$ in the stationary system, it is based on coördinates "natural" to these space-time points; in the sense that they are those of a stationary system coinciding with the rotating one at the space-time points considered. It will be noticed that for these points the curvature given by (13) is ω^2/c^2. This shows that the curvature of the spacial cross-section at any space-time point in the coördinates "natural" to this point is constant; it is the square of the angular velocity of rotation in radians per light-second.

THE ENERGY LOSSES ACCOMPANYING IONIZATION AND RESONANCE IN MERCURY VAPOR

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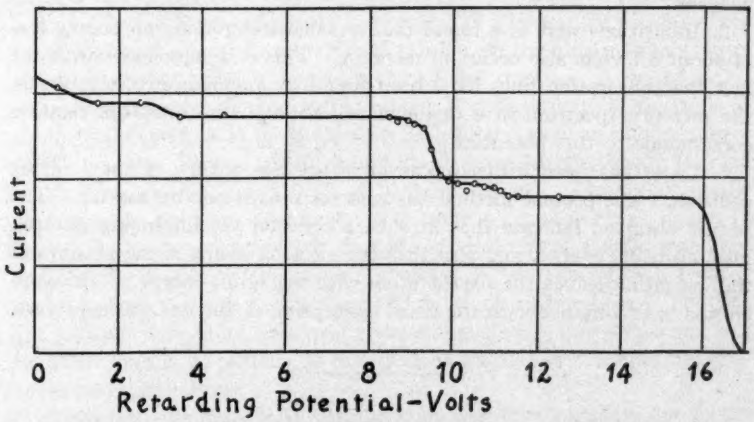
Much light has in recent years been thrown upon the constitution of matter, and upon the validity of the Bohr theory of atomic structure by the study of resonating and ionizing collisions of electrons in vapors and gases. There are, however, still a great many questions of a fundamental nature in regard to such phenomena which cannot be answered by the

experimental methods which have hitherto been employed for the determination of the resonating and ionizing potentials. It seemed desirable to use a new type of tube, with which the actual distribution of velocities of the electrons after their collisions with the vapor molecules could be determined.

Instead of the grid employed in the usual form of tube two diaphragms, placed one behind the other and each pierced with a single small hole, were used. These diaphragms divided the tube into two regions: an upper one in which any desired pressure of mercury vapor could be maintained and in which the collisions to be studied took place; and a lower region from which mercury was removed by liquid air and in which the electron velocities were measured. In the upper region the electrons were emitted from an oxide coated cathode and accelerated by a grid which



was maintained at any desired potential. Passing through the grid, the electrons entered a constant potential region and after a large number of collisions with the mercury vapor, some passed through the two small holes into the lower, mercury free region and to the receiving electrode. To this electrode was applied a variable retarding potential and by measuring the current as a function of this retarding voltage the velocity distribution of the electrons was determined.

In the curve shown the current is plotted against the retarding potential for electrons which had, before collision, velocities ranging between 15.8 and 16.8 volts. We find in the curves drops beginning at 0, 3, 9.1, 10.9 and 15.8 volts, respectively. The drop at 15.8 volts represents those electrons which after collision retained all of their energy. The other drops in the current are due to groups of electrons which have lost respectively all, 13.4 (i.e. 2×6.7), 6.7 and 4.9 volts of energy which they had

before collision. No drop is seen at 5.4 volts which would correspond to the loss of 10.4 volts which is normally assumed to take place at ionization.

From curves of such nature for different velocities of impact, it has been possible to determine more surely than heretofore the energy losses at electron impact. The outstanding results of the investigation are:

1. The discovery by Mohler, Foote and Meggers of a resonance potential involving an energy loss of 6.7 volts was confirmed. The work from which these investigators were able to deduce the existence of this critical potential seemed subject to some very serious difficulties in interpretation. These difficulties have been cleared up. The 6.7 volt type of collision does not occur for electrons with energies less than 8.5 volts, but at voltages above the ionization point it is the most important type of resonating collision.

2. Indications were also found that a collision involving an energy loss of about 5.7 volts also occurs in mercury. This is in agreement with the fact that absorption lines have been found by previous investigators in the mercury spectrum in a region which through the quantum relation corresponds to this potential.

3. Little has been hitherto known about the nature of the ionizing collision. The present method has cast some light on this matter. The results obtained indicate that at such a collision the impinging electron loses all of its energy, and also that the electron which is the product of the ionization leaves the parent atom with negligible energy. This conclusion is at variance with the usual conception of the ionization process.

THE CRYSTAL STRUCTURE OF BERYLLIUM AND OF BERYLLIUM OXIDE

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The structure of the lighter elements and of their compounds should, on account of their greater simplicity, be of especial value in deciding what rôle the so-called valence electrons play in connecting atoms. Beryllium (Be; atomic number 4) is the lightest metal the crystal structure of which has not hitherto been determined by X-ray methods. Its accepted atomic weight is $w = 9.10$, although a recent determination,[1] apparently of high accuracy, gives $w = 9.018$ which is in much better agreement with the

evidence indicating that the element consists of a single atomic species.[2]

Through the kindness of one of my colleagues, Dr. H. E. Ives, a small supply of metallic beryllium, recently prepared for him in this laboratory by Mr. G. O. Smith, was made available to me for this study. The method of preparation was by the electrolysis of $2BeF_2.NaF$ with from 7 to 15 amperes at 80 volts. This is a modification of the method of Fichter and Jablczynski[3] who used a lower voltage. The product of the process is a fine gray powder interspersed with thin flakes up to half a millimeter in greatest dimensions, which show good metallic lustre. The principal impurity to be expected is beryllium oxide (BeO), and this should not exceed in amount a few per cent by weight.

A thin-walled pyrex glass tube about one millimeter in diameter was packed loosely with the beryllium powder, the looseness of packing being intended to reduce the chance of anisotropy.[4] This sample was not thick enough to cast a very sharply defined shadow on the film, a circumstance which somewhat reduces the accuracy of the results.[5] A water-cooled molybdenum target, bombarded by about 30 milliamperes of electrons at about 30,000 volts, and provided with ZrO_2 filters[6] gave an intense source of X-rays, the photographically important wave-lengths being[7] 0.71212×10^{-8} cm. (K α_2) and 0.70783×10^{-8} cm. (K α_1). The radius of the photographic film was 20.36 cm. The time of exposure was about 18 hours. In taking the first photograph the cylindrical sample was kept stationary; in taking the second photograph the sample was rotated about its axis through a small angle at intervals of one hour. The two photographs were quite similar in general appearance, but the second had smoother lines in its pattern, so that it alone was used in the final measurements and calculations.

In order to eliminate from consideration any lines certainly due to the presence of BeO a sample of that substance in the form of a fine powder was examined in the same way. The results are presented in Table I, and show that the diffracting centers lie at the points of two symmetrically interpenetrating hexagonal space lattices, each with the axial ratio $c/a = 1.63$ and with $a = 2.696 \times 10^{-8}$ cm. There are three faint lines which must be attributed to the presence of impurities. Assuming that one molecule, BeO, is associated with each lattice point and taking[8] $N = 6.0594 \times 10^{23}$, $w = 9.018$, this gives a computed density 2.99 gm./cm^3. Both structure and density are in agreement with published values,[9] although a recent paper by Gerlach[10] contains a very different result. A careful examination of his data, however, shows that the resolving power of his apparatus was not high, and that all the lines which he reported as intense are within a short distance of positions accountable for by the structure here proposed.

Comparing the pattern obtained for Be with that obtained for BeO it was found that the first three lines in the former and possibly also the fourth were attributable to the oxide and its principal impurity. A brief search by graphical methods[11] showed that the fundamental space lattice was hexagonal, and that the fourth line of pattern might be retained. A more exact analysis gives the results presented in Table II, and shows that the diffracting points form two symmetrically interpenetrating hexagonal lattices, each with the axial ratio $c/a = 1.58$ and with $a = 2.283 \times 10^{-8}$ cm. No lines are left unaccounted for this structure, which is consistent with the hexagonal structure ($c/a = 1.5802$) determined by optical methods.[12] Assuming that one atom is associated with each lattice point, and taking N and w as given above, this gives a computed density 1.827 gm./cm^3., or about one per cent less than the best previous values.[13] The relative positions of the two component space lattices along the hexagonal axis are not determined with any accuracy, the intensities being very roughly estimated and not particularly significant.

Comparing the proposed structure for Be with those already reported for elements in the same column of the periodic table, it is clear that it belongs rather with Mg, Zn, and Cd than with Ca, Sr, and Ba, the first three being hexagonal, all with c/a greater than 1.58 and increasing with the atomic

TABLE I—Be

ESTIMATED INTENSITY	MEASURED $2r\theta$ IN CM.	COMPUTED LOG (10^4M)	ATTRIBUTED TO[1]	COMPUTED[2] LOG (10^4a)
2	4.69	0.4911	X[3]	
6	6.60	0.3437	BeO	
1	6.97	0.3201	BeO	
4	7.44	0.2921	(10.0)	0.3546
2	8.05	0.2583	(00.2)	0.3607
10	8.37	0.2416	(10.1)	0.3611
2	11.02	0.1244	(10.2)	0.3587
4	12.89	0.0583	(11.0)	0.3593
2	14.44	0.0108	(10.3)	0.3576
	[4]		(20.0)	
	[4]		(11.2)	
2	15.55	$\bar{1}$.9801	(20.1)	0.3593
	[5]		(00.4)	
	[5]		(20.2)	
1	18.33	.9128	(10.4)	0.3572
			Mean	0.3586
			Result a =	2.283×10^{-8} cm.

[1] All planes theoretically capable of reflection within the range of the tabulated values of $2r\theta$ are included.

[2] Computed for $c/a = 1.58$.

[3] X = impurity in BeO.

[4] Diffuse band not resolved. The theoretical range in $2r\theta$ is only 0.58 cm.

[5] Not observed with certainty.

number, whereas Ca, the only member of the second three which has been analyzed, has cubic symmetry.[14] The remaining element in this column, Hg, has rhombohedral symmetry.[15]

The oxide, BeO, finds an analog in ZnO, all the other monoxides in the column having cubic symmetry,[16] except HgO which forms monoclinic crystals.[17] The structure of ZnO has been reported by W. L. Bragg[18] to be hexagonal close-packing of molecules with $c/a = 1.608$ and $a = 3.22 \times 10^{-8}$ cm. Recent data obtained by Hedvall[19] but not reduced by him is in good agreement with this structure. Bragg regards the two hexagonal lattices of the proposed arrangement as determining the positions of the zinc atom-centers, and suggests that the oxygen atom-centers lie at the points of two similar lattices derived from these by displacing both of them along the hexagonal axis by about $3c/8$. The same type of structure is at

TABLE II—BeO

ESTIMATED INTENSITY	MEASURED $2r\theta$ IN CM.	COMPUTED LOG (10^4d)	ATTRIBUTED TO[1]	COMPUTED[2] LOG (10^4a)
2	4.69	0.4911	Y[3]	
1	5.43	0.4277	X	
8	6.23	0.3685	(10.0)	0.4310
6	6.67	0.3392	(00.2)	0.4301
10	7.09	0.3128	(10.1)	0.4297
1	7.57	0.2847	X	
3	9.15	0.2035	(10.2)	0.4311
8	10.90	0.1290	(11.0)	0.4300
8	11.89	0.0923	(10.3)	0.4308
	[4]		(20.0)	
6	12.86	0.0593	(11.2)	0.4303
	[4]		(20.1)	
	[4]		(00.4)	
	[4]		(20.2)	
	[4]		(10.4)	
2	16.40	$\bar{1}$.9582	(20.3)	0.4293
1	16.90	$\bar{1}$.9459	(21.0)	0.4309
1	17.29	$\bar{1}$.9366	(21.1)	0.4303
	[4]		(11.4)	
	[5]		(10.5)	
2	18.34	$\bar{1}$.9126	(21.2)	0.4304
	[4]		(20.4)	
1	19.35	$\bar{1}$.8911	(30.0)	0.4305
2	20.04	$\bar{1}$.8770	(21.3)	0.4298
			Mean	0.43032
			Result $a =$	2.696×10^{-8} cm.

[1] All planes theoretically capable of reflection within the range of the tabulated values of $2r\theta$ are included.

[2] Computed for $c/a = 1.63$.

[3] X = impurity.

[4] Not observed with certainty.

[5] Forms with next an irresolvable doublet ($\Delta s = 0.01$ cm.).

least consistent with the data for BeO. In both cases there should be different properties in the two directions of the hexagonal axis.

The fact that c/a is greater for Zn than for Be, but less for ZnO than BeO is hard to explain unless we assume a considerable rearrangement of electrons to take place during the formation of one of the oxides. It is interesting to note that another substance which possesses this structure is ice,[20] which tempts one to regard frozen water as an oxide of the formula $(OH_4)O$, the radical OH_4 being divalent, just as NH_4 is univalent and CH_4 is saturated.

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[13] C. L. Parsons, *loc. cit.*; F. Fichter and K. Jablczynski, *loc. cit.*

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[17] P. Groth, *loc. cit.*

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[20] W. H. Bragg, *Phys. Soc. London Proc.*, **34**, 1922 (98–103).

SOME CASES OF NERVE-DEAFNESS AND THEIR BEARING ON RESONANCE THEORIES OF AUDITION

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Communicated June 8, 1922

1. *Introduction.*—The present paper deals with certain cases of nerve-deafness and their bearing on the mechanical resonance theories of tone perception. The paper is thus a continuation of a recent one published in the February issue, 1922, of the *Physical Review* and like the earlier

one, is a result of further coöperation with some of the ear specialists of Chicago.

It is generally accepted among medical men that the purpose of an end-organ is to lower the threshold of perception of certain sensations. In the case of the ear, the complicated internal-ear structure is not only for the purpose of lowering the threshold of audition, but, according to all the mechanical resonance theories of pitch perception, it is also for the purpose of making possible the phenomenon of this perception. Data which have now been collected show with much certainty that these resonance theories are untenable and it is the purpose of the present paper to present some of these data. The results seem to show clearly that the sole purpose of the mechanical structure of the internal-ear is to lower the threshold of audition and that the power of pitch perception is a property of the nerves including their endings. According to this view, then, the power of pitch perception is in no way associated with the actual mechanical structure of the cochlea.

2. *Experimental Results.*—The details of the method of testing were described in the paper to which reference has already been made and these need not be discussed in this place. Many cases of nerve-deafness have been examined but only those that have a direct bearing upon the purpose in mind will be presented.

a. One of the most interesting set of curves for nerve-deafness is that shown in figure 1. This set is for a person 45 years of age. The upper curve was taken in December, 1920. The lower curve for the same ear was taken in March, 1922, so an interval of sixteen months had elapsed. The curves show that the ear functioned in a normal manner up to 1000 d. v. So, as pointed out in the previous paper, we know that no deficiency in hearing exists because of middle-ear trouble. During the period of sixteen months the impairment of hearing has become quite marked; requiring about 665,000 times as much current through the receiver for the threshold of audition at 3000 d. v. as is required by a normal ear. The minimum of the earlier curve has shifted from 3300 d. v. to 3000 d. v. during the interval. The range in frequency for which a deficiency in hearing exists includes lower pitches than were included in the first test, and, moreover, over the affected range the impairment is much more marked than indicated by the first curve. It is almost incredible that a person could be as deaf as indicated by these curves at some pitches and still have normal hearing at other frequencies.

In the case of the patient under discussion, near the threshold of audition at the higher pitches, say in the region of 3000 or 4000 d. v., the tones would break through, so to speak; they would be heard as pure tones with much intensity. A moment later it seemed as though the supply of nerve energy had been exhausted and the sensation of a pure tone would change

to a blurred noise of just audible intensity. When the tone was heard no noise sensation was present and when the noise was heard no tone was observed. So, it would seem that the same nerves were responding under both conditions and that a certain minimum amount of nerve energy was necessary for tone perception. It is quite improbable that such a variation as this could be due to the mechanical condition of the internal-ear but it would appear most reasonable to consider it due to a condition of the auditory nerve system whose impairment, without doubt, was responsible for the deficiency in hearing. Even though there was such an impairment in hearing due to the condition of the nerves and their endings the patient was able to recognize all the pitches when the tones were sufficiently intense.

b. In figure 2 is shown a group of six curves for the right and left ears of three partially deaf and dumb persons who had congenital deafness. Impaired hearing of this nature is caused, according to medical findings, by the incomplete or mal-development of the internal-ear system. These six curves are of much value from the theoretical viewpoint as will be seen later.

Curves 1 and 2 are for the two ears of a young man 25 years of age. Curves 3 and 4 are for those of a young girl 18 years of age and Curves 5 and 6 are for the two ears of a boy 10 years old. The first two curves are of the same general shape; both have their maximum depression at 3300 d. v., but the degree of hearing for the two ears is quite different. Curves 3 and 4 are also of the same general shape and the same is true of Curves 5 and 6. All three of these patients recognized all the pitches, the tones sounded of the same pitch when listened to by the two ears (that is there was present no indication of diplacusis for these cases) and the pitch discriminating power was apparently normal. Therefore, the auditory nerve systems for these three patients responded in a normal manner in regard to pitch perception. The sound intensity, of course, needed to stimulate the nerves in order to produce the sensation of sound, was far more than required for normal hearing; this being due, no doubt, to the structural condition of the cochlea.

Although these three persons were nominally deaf and dumb, all three of them recognized the difference between the fourteen fundamental vowel sounds when they were intoned in the patients ears. In the case of the girl the vowel sounds were repeated correctly by her. She was able to recognize and repeat, but with difficulty, spoken words and simple sentences. The results were not as satisfactory in the case of the boy. Nevertheless, he was able to repeat the vowel sounds with sufficient clearness to make it certain that he was able to distinguish the difference between these sounds.

c. In figure 3 are given two curves for patients each of whom were considered by the otologist totally deaf in one ear. The threshold curves

of audition showed that both patients were able to hear all the tones at sound intensities of not unusually large magnitudes, and furthermore both patients had about the same impairment of hearing. They were also able to recognize perfectly all the tones of the various pitches at the increased intensities. Care was taken to make sure that the patients were

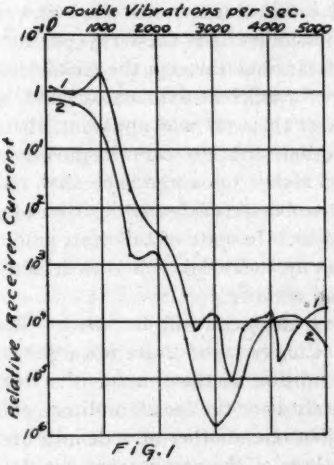

FIG. 1

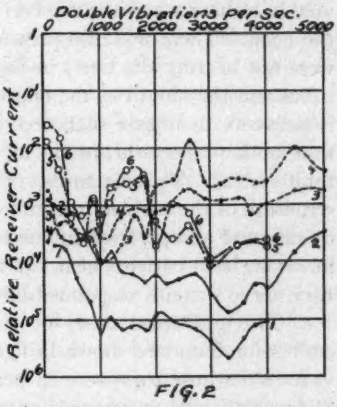

FIG. 2

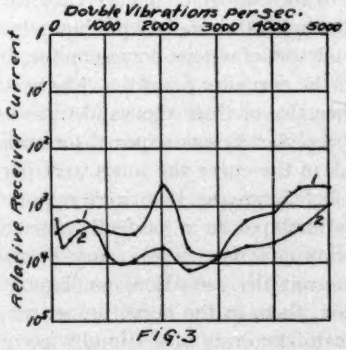

FIG. 3

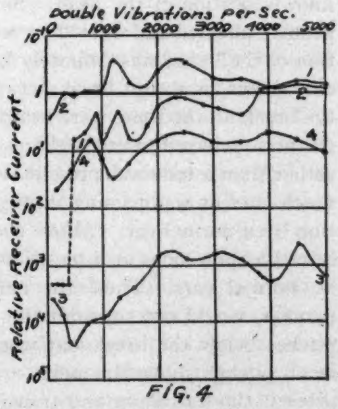

FIG. 4

hearing with the tested ears and not with the opposite ears. In the case of the patient represented by Curve 1 the opposite ear possessed normal hearing at 1000 d. v. and 4000 d. v. and it was assumed that the opposite ear was normal. The opposite ear, however, in the case of the patient represented by Curve 2 was quite defective in hearing. The possibility existed

that these patients were hearing in these opposite ears and that they were totally deaf in the other ears, as diagnosed. However, the moistened finger was placed in the ear which was not being tested and in the case of both patients the sensation of sound disappeared when the receiver was removed a cm. or so from the ear. They were able to hear when the receiver was brought close to but not in contact with the ear and they were unable to hear when the receiver was placed in contact with the bones of the head. There was also present the possibility that these two patients were not hearing the tones in the tested ears but through the eustachian tubes and the mouth to the opposite ears. Sufficient data are at hand, it is believed, to negate such a conclusion as this. It was apparent, then, that both patients heard by air conduction with the ears diagnosed as totally deaf. The diagnosis, of course, rested upon the fact that the structure of the cochlea was known to have been completely destroyed by disease and an operation in one of the cases. In spite of this destruction, however, both patients heard all the tones up to 5000 d. v. and their auditory nerve systems responded in a normal manner.

d. Two additional cases of true nerve deafness will be cited. The curves for these are shown in figure 4. Curves 1 and 2 are for a girl 20 years old whose ears were in perfect condition at the time of the test. She was affected on one side only by paralysis of the facial, auditory and vestibular nerves. These three nerves join one another at a definite and known location in the head. Since all three of the nerves were simultaneously affected and since they recovered at the same time, the exact location of the lesion was definitely known to the otologist. Curve 1 for this case shows an abrupt break between 400 and 500 d. v. At pitches below the break all the tones were heard as some sort of a noise corresponding to the noise referred to in the discussion of the curves in figure 1. The noise rather than a tone was present even when two or three thousand times as much current was passing through the receiver as was required for audition by a normal ear. Above the break in the curve the tones were perceived as pure ones at intensities only slightly greater than were required by normal ears. Thus, the nerves, stimulated in a perfectly normal manner, would not transmit the sensation of a pure tone at any of the pitches below the break but would transmit the sensations undistorted at all pitches above this point. A lesion, then, in the nerve proper may prevent the formation and transmission to the brain of a stimulus corresponding to a pure tone.

This patient was first tested on February 4, 1921, and was again tested on March 7, 1921. During this period she had recovered completely and her hearing as shown by Curve 2 in figure 4 was equivalent to or better than normal. So, with the disappearance of the cause of the paralysis there was an immediate return to normal hearing and pitch perception.

The concluding case to be cited was also one of pure nerve lesion due to pressure on the auditory nerve brought about by the growth of a cerebral tumor. Curve 3, figure 4, shows the relative degree of hearing possessed by the patient's left ear. The right ear was not greatly affected; as shown by Curve 4 it required for minimum audition a relative receiver current of 48.5 at 200 d. v. and improved with increasing pitch, requiring a relative receiver current of 11.5 at 5000 d. v. The curve (4) for the right ear, however, is almost exactly of the same shape as that (3) for the left ear. The lesion, then, had affected both ears alike qualitatively. In the case of this patient it has not been found possible to secure additional curves.

These two cases of true nerve deafness show the maximum depression in hearing at the lower frequencies, whereas for most cases of what is termed nerve deafness caused by lesions within the internal-ear the greatest depressions occur at the higher pitches.

Many additional curves have been taken on patients with nerve deafness, but for the purpose of the present paper it is believed that the cases cited are sufficient to establish the object in view.

3. *Theoretical Bearing of the Cases Cited on the Mechanical Theories of Tone Perception.*—As stated in the introduction, it is believed that the data presented herewith show conclusively that the sole purpose of the mechanical structure of the internal-ear is simply to lower the threshold of audition and that mechanical resonance of this structure is not in the slightest degree responsible for the phenomenon of tone perception. The group of curves shown in figure 3, for patients known to have internal-ears completely modified, showed normal pitch perception but greatly reduced sensitivity. If mechanical resonance of this structure were responsible for tone perception, then this perception would have been completely destroyed in these two cases. This sense remained unmodified, but the greatly decreased sensitivity of hearing would suggest conclusively that the internal-ear structure is for the prime purpose of lowering the threshold of audibility.

This view is much strengthened by the group of six curves in figure 2 for deaf-mutes all of whom showed simply reduced sensitivity of hearing without any apparent modification of the sense of tone perception. Among medical men it is considered that congenital deafness of this sort means that the cochlea is only partially developed. Accepting this view as even partially correct, we are forced to the conclusion that all theories of audition which are based on mechanical resonance of the internal-ear are untenable.

The data for Curve 1 are the same as given in figure 17 of the paper published in the February 1922 issue of the *Physical Review*, p. 88. The argument against the mechanical resonance theories as stated in this earlier paper, pp. 95–96, can be repeated here with even more decisiveness than

was given there because the patient has become more deaf at the higher pitches as shown by Curve 2, figure 1, since the first curve was taken.

It would seem, therefore, even in the case of the complete modification of the internal-ear mechanism that a person can hear in a normal manner, provided the tones are sufficiently intense and provided there has not been a complete destruction of the nerve endings or of the nerves themselves in which case we should expect complete deafness. Only one such case has as yet been found and in this case the patient was unable to hear at any pitch in either ear with as much as 5×10^6 times as much current passing through the receiver as was required by a normal ear.

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THE QUANTITATIVE INFLUENCES OF CERTAIN FACTORS INVOLVED IN THE PRODUCTION OF CYANOSIS

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Lundsgaard has shown that the appearance of cyanosis depends on the mean concentration of reduced hemoglobin (C) in the capillary blood. This concentration he estimated as

$$C = \frac{A + V}{2} \qquad (1)$$

where A is the arterial, V the venous concentration of reduced hemoglobin. The effect of certain physiological factors contributing to C is estimated as follows. We let T represent the total hemoglobin concentration in the blood, l the fraction of total hemoglobin passing in reduced form through the aerated parts of the lungs, D the concentration of reduced hemoglobin formed by deoxygenation of the blood as it passes from arteries to veins through the tissue capillaries. In certain pathological conditions a fraction of the venous blood reaches the arteries without traversing aerated parts of the lungs and therefore carries reduced venous hemoglobin directly into the arterial blood. This fraction we designate as α. A, T, and C have the significance indicated above.

When $\alpha = O$, only aerated blood entering the arteries, $A = l\,T$. When however, α has a positive value, the reduced hemoglobin of the arterial blood represents the sum of that in the fraction α, of venous blood entering

the arteries, plus that of the fraction $(1 - \alpha)$ of arterial blood from the lungs. In such a case

$$A = (1 - \alpha)\, l\, T + \alpha V \tag{2}$$

$$\alpha = \frac{A - l\, T}{V - l\, T} \tag{3}$$

Substituting $V - D$ for A in (3) and solving for V,

$$V = 1\, T + \frac{D}{l - \alpha} \tag{4}$$

Substituting $A + D$ for V in (3) and solving for A,

$$A = l\, T + \frac{\alpha D}{1 - \alpha} \tag{5}$$

We now substitute for A and V in (1) their values as expressed in (4) and (5), and obtain

$$C = l\, T + \frac{(1 + \alpha)D}{2(1 - \alpha)} \tag{6}$$

which indicates the effects of the four factors, l, T, α, and D on C.

An interesting point brought out by (6), and in accord with observation, is that other factors normal about 40 per cent of venous blood may be mixed with arterial ($\alpha = 0.4$) before the reduced hemoglobin concentration in the latter becomes great enough to cause cyanosis.

The effects on C of separate variations of l, T, D, and α are indicated by partial differentiation of (6).

$$\partial C = T\, \partial l \tag{7}$$

$$\partial C = l\, \partial T \tag{8}$$

$$\partial C = \frac{D}{(1 - \alpha)^2}\, \partial \alpha \tag{9}$$

$$\partial C = \frac{(1 + \alpha)}{2(1 - \alpha)}\, \partial D \tag{10}$$

While the reduced hemoglobin concentration in the capillary blood may be accepted as the cause of cyanosis, there are various other factors which modify the resulting coloration. Such are local skin vascularity, pigmentation, thickness of epidermis; and also the fact that the mean capillary content of reduced homoglobin $\frac{1}{2}(A + V)$ only approximately represents the average content. With changing conditions the latter may instead of being midway between venous and arterial, approximate either more

nearly than the other. The effect of these modifying factors is to cause the value of C at which cyanosis becomes perceptible to vary from 4 to 6 gms. of reduced hemoglobin per 100 cc. of blood, and perhaps sometimes even more widely, although it appears usually to lie near 5.